PHYS 331 – Assignment #1

Due Tuesday, October 3 at 09:30 am

1. Note that formal reports for experiment #1 will be due Oct. 30. You should start writing. Of course, you don't have all of the data that you will analyze and present, but you can always start writing your introduction and theory sections. You may also be able to begin to describe the data analysis methods.

Find and list 4 references that can be used for your project. None can be websites and only one can be a textbook. None can be the material that was provided to you on the Canvas website. In a few short sentences, describe why each reference will be a valuable resource for your project.

Continued on the following pages...

In the seminar we showed that lock-in amplifiers can be used to determine the amplitude and phase of a signal response to a sinusoidal stimulus.

Specifically, we assumed that the desired experimental signal was of the form $v_{\mathrm{S}}(t) = A_{\mathrm{S}} \sin(\omega_{\mathrm{S}} t + \phi)$ and the stimulus, or reference signal, was of the from $v_{\mathrm{R}}(t) = A_{\mathrm{R}} \sin(\omega_{\mathrm{R}} t + \theta)$. The lock-in amplifier multiplies $v_{\mathrm{S}}(t)$ and $v_{\mathrm{R}}(t)$, passes the product through a low-pass filter, and then the averages the output of the filter. For all $v_{\mathrm{S}}(t)$ for which $\omega_{\mathrm{S}} \neq \omega_{\mathrm{R}}$, the lock-in output averages to zero. When $\omega_{\mathrm{S}} = \omega_{\mathrm{R}}$, however, the filtered and averaged lock-in output is given by:

$$\langle v_{\mathrm{S}}(t) v_{\mathrm{R}}(t) \rangle_{\omega_{\mathrm{S}}=\omega_{\mathrm{R}}} = \frac{A_{\mathrm{S}} A_{\mathrm{R}}}{2} \cos(\phi - \theta).$$

The experimental signal has two unknowns (A_{S} and ϕ) such that two measurements are needed to completely characterize it.

The first measurement is made with $\theta = 0$ and the second with $\theta = \pi/2$ such that:

$$\langle v_{\mathrm{S}} v_{\mathrm{R}} \rangle_{\omega_{\mathrm{S}}=\omega_{\mathrm{R}}} |_{\theta=0} = \frac{A_{\mathrm{S}} A_{\mathrm{R}}}{2} \cos\phi = \frac{A_{\mathrm{R}}}{2} X$$

and

$$\langle v_{\mathrm{S}} v_{\mathrm{R}} \rangle_{\omega_{\mathrm{S}}=\omega_{\mathrm{R}}} |_{\theta=\pi/2} = \frac{A_{\mathrm{S}} A_{\mathrm{R}}}{2} \sin\phi = \frac{A_{\mathrm{R}}}{2} Y$$

where $X \equiv A_{\mathrm{S}} \cos\phi$ and $Y \equiv A_{\mathrm{S}} \sin\phi$.

In this case the signal amplitude and phase can be determined via:

$$A_{\mathrm{S}} = \sqrt{X^2 + Y^2}$$

and

$$\tan\phi = \frac{Y}{X}.$$

In some cases the experimental signal may be a nonlinear function of a sinusoidal stimulus. For example, the absorption peak in an ESR experiment is a nonlinear function of the magnetic field strength. So, if the strength of the magnetic field is varied sinusoidally (with a small amplitude) about its average value, the absorption signal will oscillate approximately sinusoidally and this oscillation can be detected using a lock-in detector. In this assignment, we attempt to show that a lock-in amplifier can be used to determine the first derivative of a system's response to stimulus $x(t)$. We saw how this works intuitively in the lecture by drawing a series a pictures. You will come to the same conclusions in this assignment via a more mathematical argument.

2. Assume that the signal to be fed into a lock-in detector is of the form:

$$v(t) = v(x(t))$$

and

$$x(t) = \overline{x} + x_{\mathrm{ac}}(t) = \overline{x} + A\sin(\omega t)$$

where x is the stimulus. x has an average dc value of $\overline{x}$ and it's value is varied sinusoidally about that average value. For example, v could be the absorption signal in an ESR experiment and $x(t)$ the strength of an external magnetic field.

Let's take a specific example:

$$v(t) = [x(t)]^n = [\overline{x} + A\sin(\omega t)]^n .$$

We will attempt to show that, if $A \ll \overline{x}$, the lock-in amplifier can be used determine a quantity that is proportional to $dv/dx|_{\overline{x}}$.

To do this, there's one more thing you should know about lock-in amplifiers: they first use a high-pass filter to block any DC component from the reference and signal inputs before multiplying them and then applying the low-pass filter and averaging.

(a) For the specific case we're considering, show that, after high-pass filtering, the signal to be processed by the lock-in detector is given by:

$$v_{\mathrm{S}} \approx n\overline{x}^{n-1} A\sin(\omega t) = \left.\frac{dv}{dx}\right|_{\overline{x}} A\sin(\omega t) .$$

(b) If $v_{\mathrm{R}} = A_{\mathrm{R}}\sin(\omega t + \theta)$, then what would be the values of X and Y which were defined on the previous page?

(c) What would be the expressions $\sqrt{X^2 + Y^2}$ and Y/X yield? Give your answer for $\sqrt{X^2 + Y^2}$ in terms of dv/dx.